MA/MSCMT-04

June - Examination 2016

M.A./M.Sc. (Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Section 'A' contain (08) Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write equation of osculating plane in terms of generating parameter 't'.

(ii) Define Torsion.

(iii) Define Indicatrix.

(iv) Write criterion for a surface to be developable.

(v) Write first fundamental form.

(vi) Write the relation between three fundamental form.

(vii) Write differential equation of geodesics in a V_N.

(viii) State Gauss's characteristics equation.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Section 'B' contain Short Answer Type Questions. Examinees have to answer **any four** (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that curvature and torsion of either associate Bertrand curves are connected by a linear equation.

3) Find the equation to the conoid generated by lines parallel to the plane XOY are drawn to intersect OZ and the curve $x^2 + y^2 = r^2$; $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}$

4) Prove that the envelope of a family of surfaces touches each member of the family at all points of its characteristic.

5) Show that for the right helicoid $\vec{r} = \left(u \cos v, u \sin v, cv\right)$

$$l = 0, m = 0, n = -u; \lambda = 0, \mu = \frac{u}{(n^2 + c^2)}, v = 0$$

6) If A^{ijk} is a skew-symmetric tensor; show that

$$A^{ijk}_{ji} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j}\left(A^{ijk}\sqrt{g}\right)$$

7) Obtain differential equations of geodesics for the metric

$$ds^2 = f(x)\,dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}\,dt^2$$

8) Contract the Riemann Christoffel tensor and find Ricci Tensor.

9) If the metric of a two dimensional flat space is $f(r)\left[\left(dx^1\right)^2 + \left(dx^2\right)^2\right]$ where $(r)^2 = (x^1)^2 + (x^2)^2$; show that $f(r) = c\,(r)^k$ where c and k are constants.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer **any two** (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove theorem related to geometrical significance of second fundamental form and derive Weingarten equations.

11) Find the principal sections and principal curvatures of the surface $x = a(u + v)$, $y = b\,(u - v)$, $z = uv$

12) State and prove Gauss - Bonnet theorem.

13) (i) Show that the metric of a Euclidean plane referred to cylindrical co-ordinates is given by

$$ds^2 = dr^2 + (rd\,\theta)^2 + dz^2$$

(ii) Show that Divergence of Einstein Tensor Vanishes.